

1201. Insert the output $f(x)$ as the input of f . Simplify: multiply top and bottom of the main fraction by the denominator of the inlaid fraction.

1202. The equation of the line is

$$y = a + \frac{b-a}{h}x.$$

Integrate this definitely between $x = 0$ and $x = h$.

1203. Consider common factors.

1204. Construct such a value by dividing e.g. π by a large number, and adding it to 3.6.

1205. (a) Write $2x + 7 \equiv 2(x + 3) + 1$.

(b) Consider the graph as $y = \frac{1}{x}$ transformed.

1206. Since no two lines are parallel, every pair of lines must cross exactly once. Work out how many pairs of lines there are.

1207. (a) Use the proposed linear solution, not the DE.

(b) Substitute for $\frac{dy}{dx}$ and y in the DE.

(c) The formula in (b) is an identity. So, equate the coefficients of x , and the constant terms.

1208. Rearrange to the form $f(x) = 0$ and factorise.

1209. (a) Draw a plan view of $ABCD$, marking the forces as points: the forces are perpendicular to the square. Consider the symmetry of the diagram.

(b) Set up equations: equilibrium perpendicular to $ABCD$ and moments about the perpendicular bisector of BC .

1210. Consider the equation of a circle.

1211. (a) Consider the fact that $x = k$ is a vertical line, parallel to the y axis.

(b) Use the discriminant.

1212. Express the relationship between a , the side length, and b , the diagonal length, as an equation, and differentiate it with respect to time.

1213. Draw a sketch and count the points.

1214. (a) Differentiate f .

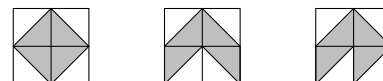
(b) Put 0 in your calculator and press enter. Then type the iteration with ANS in place of x_n . Pressing enter repeatedly runs the iteration.

(c) Quote the factor theorem.

(d) Take out the linear factor. If you need to, use polynomial long division. Then show that the discriminant of the resulting quadratic factor is negative.

1215. Sketch $y = 2^x$ and $y = 4^x$, which are standard exponential growth graphs. Then translate them, according to an input transformation.

1216. (a) There are three cases:



By considering transformations (reflection and rotation), give the number of outcomes in each.

(b) Use $p = \frac{\text{successful}}{\text{total}}$.

1217. Enact the operation “differentiate with respect to x ”, as encoded in the differential operator $\frac{d}{dx}(\)$, and rearrange.

1218. (a) The acceptance region is the complement of the critical region. The relevant point is the form of $P(a < \bar{X} < b)$.

(b) What is the probability of the critical region?

1219. Simplify the ratio $\frac{w_{n+1}}{w_n}$.

1220. The statement is false.

1221. One of the statements is true. The counterexample to the other two is if f and g are the same function.

1222. Write the integral as $\int_1^2 x^{-2} + 2x^{-3} dx$.

1223. The modern use of the word *reaction* is not the same as in Newton’s “action and reaction”. We now use “reaction” to refer to any contact force acting perpendicular to the surfaces in contact.

1224. Draw a line joining the two points of intersection. Calculate the area of one segment, by expressing it as a sector minus a triangle. Double this value to find the area of the shaded region. The unshaded region is one full circle minus the shaded region.

1225. Consider the numbers as a sequence with first term 1 and common difference 2.

1226. Since the functions share no roots, the factors can be taken separately: the numbers of roots then add. In (c), the reciprocal of a function can never be zero.

1227. The region is a right-angled quadrant bounded by two straight lines.

1228. Use $s^2 = \frac{\sum x^2 - n\bar{x}^2}{n}$
1229. Consider a scale factor acting on $a + b$.
1230. Assume, for a contradiction, that a pair of distinct circles has three distinct intersections P, Q, R . Consider the perpendicular bisectors of two pairs of these: show that the circles are not distinct.
1231. (a) Draw a force diagram, find the acceleration, and then use $s = ut + \frac{1}{2}at^2$.
 (b) Once the fuel is exhausted, the rocket can be modelled as a projectile. Remember that it has non-zero initial velocity for the projectile stage.
1232. Draw the mutual centre of the circles in, and set up a $(30^\circ, 60^\circ, 90^\circ)$ triangle, two of whose sides are radii, one of C_1 and one of C_2 .
1233. (a) A_n has ordinal formula $A_n = pn^2 + qn + r$, for some $p, q, r \in \mathbb{R}$ with $p \neq 0$.
 (b) B_n has ordinal formula $B_n = pn + q$, for some $p, q \in \mathbb{R}$, with $p > 0$.
1234. Consider the two possibilities BR and RB.
1235. Consider the two transformations which take the first curve onto the second.
1236. For such identities to be true, the relevant graph, e.g. $y = \cos x$, has to have the y axis as a line of symmetry.
1237. Draw a force diagram of the block, and set up two equations: vertical $F = 0$ and moments around a sensible point (one of the supports, say).
1238. You can't take the limit as it stands. Look for common factors on the top and bottom first.
1239. Carry out the factorisation explicitly; you might want to use a polynomial solver to help.
- ALTERNATIVE METHOD —————
- Find the roots of the quadratic and use the factor theorem.
1240. (a) Use interval set notation, with $[$ to include 1 and $)$ to exclude 10.
 (b) "Linear in b " means expressible as $mb + c$, for some constants m and c .
1241. The discriminant is $\Delta = b^2 - 4ac$.
1242. A rhombus has four sides of equal length.
1243. One of these is a length, the other three are areas.
1244. Set $x = 2$ to find b . Then factorise the RHS.
1245. Find the height of the ladder as a surd, then square the surd in order to compare it with 2.
1246. Expand the brackets before differentiating.
1247. Consider $\mathbf{a} = \mathbf{i}$, $\mathbf{b} = \mathbf{i} + \mathbf{j}$, $\mathbf{c} = \mathbf{j}$.
1248. Put the logarithms on one side, and use log rules.
1249. You can translate the function F as answering the question "What is the vertex of this parabola?"
 (a) Find the vertex of the parabola.
 (b) Find the equation of the monic parabola whose vertex is at $(3, 0)$.
1250. Find the distance from the centre of the equilateral triangle to one of its vertices, using trigonometry.
1251. This is the sum to infinity S_∞ of a GP. It can also be evaluated as a recurring decimal.
1252. Use the sine area formula $A = \frac{1}{2}ab \sin C$.
1253. Translate to the equation $\frac{1}{x} = \ln x$, and solve using Newton-Raphson.
1254. Consider whether $x = 1$ and $x = -1$ are single or double roots.
1255. Square both sides of the first equation.
1256. Consider the number and multiplicity of the roots of the quartic $x^4 - x^2 - 6 = 0$.
1257. (a) Subtract the second equation from four copies of the first equation.
 (b) Use the second and third equations to show that $2a - c = -3$.
 (c) Solve the equations from the first two parts simultaneously for a and c . Then substitute back in for b .
1258. Draw a force diagram of the entire scrum, with D_1 and D_2 as the driving forces exerted on the scrum. Calculate the acceleration using $suvat$, then use $F = ma$.
1259. With ordinal formula $u_n = an^2 + bn + c$, the second difference is $2a$.

1260. “Conditioned on X ” means that the primary branches should be X and X' ; the secondary branches should then be Y and Y' . Calculate probabilities using

$$\mathbb{P}(A | B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

1261. Proof by exhaustion is checking all possibilities. You only need check the single-digit numbers.
1262. Multiply up by the denominator of the fraction, and gather the \sqrt{x} terms. Take out a factor of \sqrt{x} .
1263. Multiply out the LHS and RHS individually; prove that they simplify to the same expression.
1264. Complete the square for x and for y . This will allow you to find the radius of the circle.
1265. Consider an element of $B \setminus A$, i.e. some x that is in B but not in A .
1266. Express 4 as 2^2 and use index laws. You’re looking for a result of the form $4^{2x+3} \equiv a(2^x)^b$.
1267. “No scores are odd” is “all scores are even”.
1268. With inlaid fractions, begin by multiplying top and bottom of the main fraction by the denominator(s) of the inlaid fraction(s).
1269. Differentiate the proposed curve to find $\frac{dy}{dx}$. Then substitute y and $\frac{dy}{dx}$. Solve for c , noting that what you have should be an identity in x .
1270. (a) Resolve vertically and solve for t .
(b) Find the horizontal and vertical velocities at the point of landing. Then use arctan.
1271. (a) The area can be calculated.
(b) The perimeter can’t.
1272. Use the binomial expansion to multiply out fourth power. Simplify and solve. Remember, since there are square roots involved, to check the validity of any roots you find.
1273. Because every diagonal lies inside the polygon, you can draw diagonals from one vertex to the others. Split the n -gon up into $n - 2$ triangles.
1274. Show that $(x - 3)$ is not a factor of the numerator, by using the factor theorem.

1275. Use 3D Pythagoras.
1276. The slash is “set minus”. So, picture a number line with all z between -5 and $+5$, with $[2, \infty)$ removed.
1277. Translate into algebra and solve: “The difference between the first and second terms is equal to the difference between the second and third terms.”
1278. (a) Use log rules.
(b) Use part (a), remembering that the logarithm function has a restricted domain.
1279. Expand the two brackets in the numerator using the binomial expansion. Simplify, divide top and bottom by h , then take the limit.
1280. (a) It’s a positive parabola.
(b) Multiply out $y = a(x - k)(x + k)$.
(c) Consider the y intercept.
1281. Draw a possibility space diagram (6×6 table), and use the information “at least one shows a four” to restrict the possibility space.
1282. (a) In the first quadrant, the curves intersect at $(0, 0)$ and $(1, 1)$.
(b) Use integration.
1283. $2x^2 - x + 1$ doesn’t have real roots, so you can’t use the (real) factor theorem. Instead, assume, for a contradiction, that
- $$(2x^2 - x + 1)(ax^2 + bx + c) \equiv 2x^4 - 3x^3 + 2.$$
- Equate coefficients until you pinpoint a problem.
1284. Use the angle in a semicircle theorem.
1285. X is a binomial distribution with $n = 100$ and $p = \frac{1}{2}$; Y is a normal distribution with $\mu = 50$ and $s = 5$. Use a calculator to find the relevant probabilities.
1286. Use the factor theorem: if a polynomial $f(x)$ has a factor of $x - b$, then $f(b) = 0$.
1287. Rotate the inner square so that its sides are at 45° to those of the outer square, and make it as big as possible.
1288. At $t = -1$, the coordinates are $(p - \cos \theta, q - \sin \theta)$. Find the coordinates at parameter $t = 1$, and then use Pythagoras.

1289. Differentiate with respect to u . Then reciprocate and square both sides. Use a Pythagorean trig identity.
1290. (a) Integrate both sides.
(b) A log rule has been misused.
1291. It doesn't matter where vertex A is, so place it anywhere, wlog. Then consider the placement of vertex B .
1292. An AP is symmetrical about its mean. Consider the mean of the four angles of a quadrilateral.
1293. (a) Differentiate, and set $a = 0$.
(b) Integrate between $t = 0$ and $t = \frac{16}{9}$.
1294. If points are equidistant from the centre of a circle, then they are equidistant from the circle.
1295. A unit vector has length 1. So, point A lies on the unit circle $x^2 + y^2 = 1$.
1296. A sign change at $x = 1$ means a sign change in the value of the expression as the value of x passes 1. Consider the parity (oddness/evenness) of the indices.
1297. Exponentiate both sides, and use index laws.
1298. Place the circle of radius 20 such that its diameter passes through the centres of the two large circles.
1299. Equate the differences to produce two equations: $u_2 - u_1 = u_3 - u_2$ and $u_3 - u_2 = u_4 - u_3$. Solve these simultaneously.
1300. Begin with $n^3 + (n + 1)^3 + (n + 2)^3$ and simplify.

————— END OF 13TH HUNDRED —————